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On vortex shedding from a circular cylinder in the critical Reynolds number régime

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The flow around a circular cylinder has been examined over the Reynolds number range 10^5 to $7{\cdot}5 \times 10^5$, Reynolds number being based on cylinder diameter. Narrow-band vortex shedding has been observed up to a Reynolds number of $5{\cdot}5 \times 10^5$, i.e. well into the critical régime. At this Reynolds number the Strouhal number reached the unusually high value of 0·46. Spectra of the velocity fluctuations measured in the wake are presented for several values of Reynolds number.

1. Introduction

At Reynolds numbers above 10000 the flow around a circular cylinder can be separated into at least four different régimes: subcritical, critical, supercritical and transcritical (Roshko 1961). The lowest régime, the subcritical, continues to a Reynolds number, R, of about 2×10^5. In this régime the boundary layers separate in a laminar state at about 80° from the forward stagnation point, and the early separation induces a high drag coefficient of about 1·2 with an associated Strouhal number of about 0·2. The drag coefficient, C_D, falls rapidly with increasing R in the critical régime to a minimum value of around 0·2. It has now been clearly established (see, for example, Tani 1964) that the formation of laminar separation bubbles are responsible for the low values of C_D. The measured values of C_D in this régime are widely scattered because the flow around the cylinder is very sensitive to small degrees of free stream turbulence and surface roughness. The only shedding frequency measurements are the early results of Relf & Simmons (1924) and one of the aims of this investigation was to obtain much more detailed measurements of shedding frequency throughout the critical Reynolds number range.

The supercritical régime is characterized by a less regular vortex shedding behaviour and it was at one time thought that this flow régime extended to extremely high Reynolds numbers. It was only with the experiments of Roshko (1961) that it was realized that a further narrow-band vortex shedding régime, the transcritical, existed. Roshko suggested that in the transcritical régime there were no laminar separation bubbles and that separation was purely turbulent. It came as a surprise that regular vortex shedding should recommence but, since vortex shedding is a result of the basic instability that exists between two shear layers and is inherent in most separated flows behind two-dimensional bodies, it is perhaps more surprising that regular shedding should ever cease. The experiments described in this paper were designed to try and discover how and when regular shedding ceases.

2. Experimental arrangement

The experiments were conducted in a 9 ft. (2·74 m) × 7 ft. (2·13 m) wind tunnel. The tunnel is of the closed return type and has a free stream turbulence level of about 0·2 % and a top speed of around 200 ft./s (61 m/s).

The cylinder used in the investigation was 7 in. (17·8 cm) in diameter and spanned the vertical 7 ft. dimension of the tunnel. The surface was coated with several layers of phenoglaze and a highly polished finish obtained. During the experiments the cylinder was frequently cleaned to ensure that it was free of dust particles. A ring of pressure tappings was inserted at mid-span at $22\frac{1}{2}°$ intervals and the cylinder could be rotated in order that the pressure at all angular positions could be measured. In addition, pressure tappings were inserted along the back of the cylinder to obtain some measure of the uniformity of the flow along the span. Measurements of velocity fluctuations in the wake of the cylinder were made with a DISA linearized constant-temperature hot-wire anemometer. The output of the hot-wire was recorded on a tape recorder. The signals were later digitized and power spectral analysis was performed on a KDF 9 computer.

3. Experimental results

3.1. *Measurements of base pressure and drag coefficient*

In order to be able to work in the critical Reynolds number régime the cylinder diameter was of necessity large and the blocked area was about $6\frac{1}{2}$ % of the total area of the working section. In the critical régime the flow conditions around the cylinder are changing rapidly with Reynolds number and it is doubtful whether any correction method can completely compensate for the effects of blockage. It was thought that the method of Allen & Vincenti (1944) was the most appropriate to use over the complete Reynolds number range examined. The corrections used are similar to those used by Roshko (1961).

The distribution of the base pressure coefficient, $(C_p)_b$, (i.e. the pressure at $\theta = 180°$) along the span of the cylinder at a subcritical Reynolds number of $R = 2 \times 10^5$, is shown in the upper curve of figure 1. The influence of the end-wall boundary layers on the distribution of $(C_p)_b$ appears to be restricted to those parts of the cylinder adjacent to the walls. $(C_p)_b$ varies by about ± 3 % along the length of the cylinder. Distributions of $(C_p)_b$ are also shown for Reynolds numbers of $3{\cdot}68 \times 10^5$ and 4×10^5. At these higher Reynolds numbers the influence of the walls encroaches further along the cylinder. The curve in figure 2 shows the variation of base pressure coefficient with Reynolds number. The interesting feature of this plot is the discontinuity recorded at $R \approx 3{\cdot}4 \times 10^5$. This was caused by a laminar separation bubble forming on one side of the cylinder only. The distribution of base pressure along the span suggested that the establishment of a bubble on one side takes place along the complete length of the cylinder

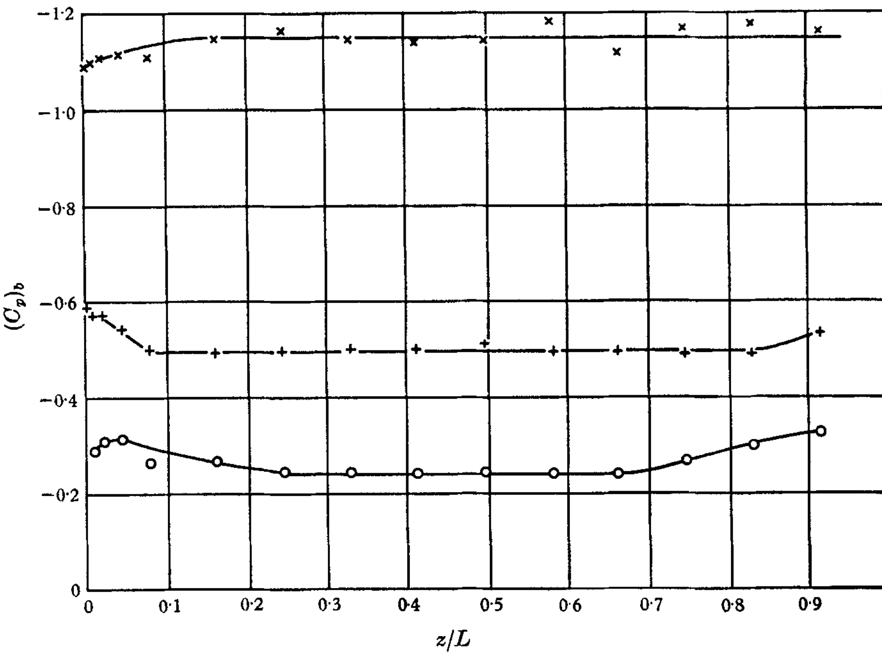

FIGURE 1. Variation of base pressure along span (z is distance from roof of tunnel and L is span) where: $\times$, $R = 2 \times 10^5$; $+$, $R = 3{\cdot}68 \times 10^5$; $\bigcirc$, $R = 4 \times 10^5$.

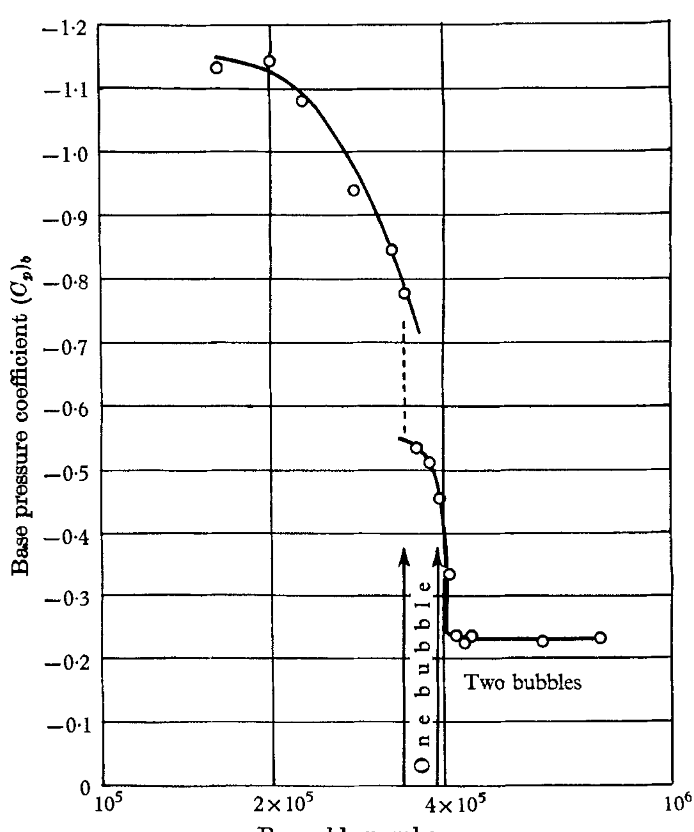

FIGURE 2. Variation of base pressure coefficient with Reynolds number.

and this was later confirmed by surface oil-flow patterns. There were no particularly obvious asymmetries in either the tunnel flow or of the model geometry to suggest a reason why the bubble formed consistently on the same side. During this phase the lift coefficient C_L, calculated from the pressure distribution, was about 1·3. At $R \approx 3{\cdot}8 \times 10^5$ the bubble on the other side formed and $(C_p)_b$ rose to the typical critical value of $-0{\cdot}23$.

R	$-(C_p)_b$	C_D	C_L	Flow pattern
2×10^5	1·14	1·14	0	No bubble
$3{\cdot}7 \times 10^5$	0·511	0·45	1·3	One bubble
4×10^5	0·235	0·23	0	Two bubbles

TABLE 1

The pressure drag coefficient, C_D, was calculated from the pressure distribution for three representative Reynolds numbers. These values of C_D are shown in table 1. At all other Reynolds numbers only base pressure was recorded. For the purposes of estimating blockage corrections, for cases where only base pressure coefficient was measured, it was assumed that the uncorrected drag coefficient was equal numerically to the measured base pressure coefficient. It can be seen that for the three cases considered above this was approximately correct.

3.2. *Measurement of shedding frequency*

It was thought that the strongest indications of shedding might be achieved by observing the output of a hot wire in the wake of the cylinder rather than one near the surface. The clearest indications were found with the wire one diameter from the rear of the cylinder and half a diameter from the wake centre line. At most Reynolds numbers the shedding frequency was sufficiently narrow band for measurements to be made accurately on a wave analyzer. Figure 3 shows Strouhal number plotted against Reynolds number. On entering the critical region the Strouhal number S began to rise as previously shown by Relf & Simmons (1924). The rise in S was greater than that found by Relf & Simmons and the scatter on the results was considerably less. Regular shedding occurred in the one bubble Reynolds number range at a Strouhal number of about 0·32 and in the two bubble range at the high value of 0·46. The only previously reported Strouhal numbers of this magnitude are those of Delany & Sorensen (1953) at Reynolds numbers in the supercritical régime where the shedding frequency is far less regular. Unlike the results of Delany & Sorensen those reported here are supported by power spectral density measurements in the neighbourhood of the Strouhal peak.

3.3. *Measurement of power spectral density*

The hot-wire signals recorded in the wake of the cylinder were digitized and power spectral density analysis was performed on a computer by the method of Blackman & Tukey (1958). The digital power spectral density analysis techniques used are described by Bearman (1968). Five spectra of velocity fluctuations in the wake at a position one diameter from the rear of the cylinder and one half diameter from the wake centre line are shown in figure 4. The spectra are presented in a non-dimensional form with $F(n)/UD$, where $F(n)$ is the power spectral density at frequency n, U is free stream velocity and D is cylinder diameter plotted against Strouhal number. The spectra are uncorrected for wind tunnel

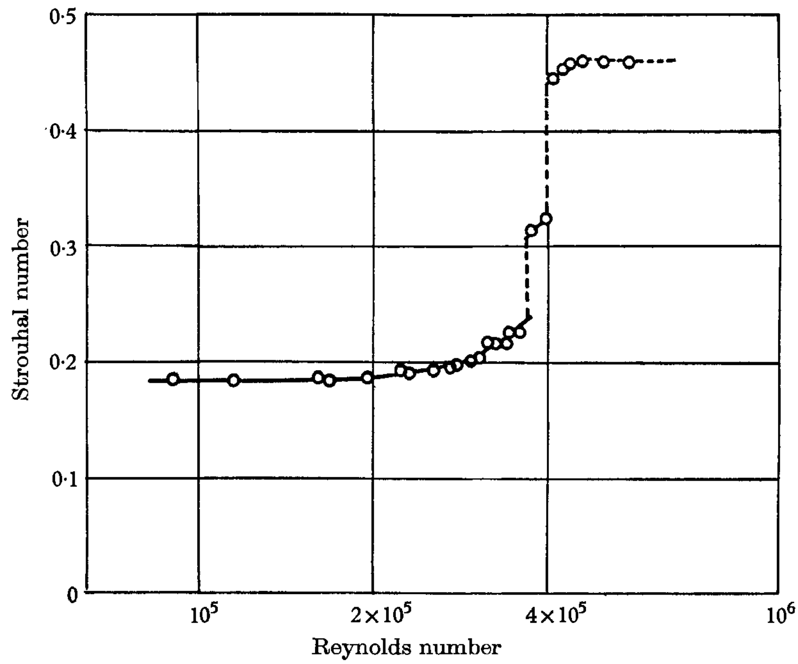

FIGURE 3. Variation of Strouhal number with Reynolds number.

blockage. This plot clearly shows the increase in the Strouhal number with increasing Reynolds number. It also indicates the extreme sharpness of the shedding peak at the Reynolds number, $4{\cdot}16\times10^5$, where the C_D is a minimum. To give a measure of the sharpness of the peak the bandwidth at the half power point Δn divided by the Strouhal frequency has been calculated. At a subcritical Reynolds number, $2{\cdot}24\times10^5$, $\Delta n/n \approx 0{\cdot}035$, whereas at $R = 4{\cdot}16\times10^5$, $\Delta n/n \approx 0{\cdot}026$.

Although the frequency is well defined in the two bubble régime the power at the Strouhal frequency is $33\,db$ down on the power at a typical subcritical Reynolds number. This suggests that any fluctuating pressures, associated with shedding, felt on the surface of the cylinder would be very much weaker. Some of the reduction in the power spectral density level will be due to the fact that the wake width is considerably smaller at these high Reynolds numbers. At this Reynolds number only very weak velocity fluctuations at the shedding frequency were observed near the surface of the cylinder at $\theta = 90°$. A hot-wire probe was positioned just outside the boundary layer on the cylinder at $\theta = 90°$ and the resulting signal is shown compared with the signal from the hot wire downstream in figure 5(*a*). The traces in figure 5(*b*), taken from hot wires on either side of the wake, illustrate the regularity of the shedding signals. In figure 4 one spectrum is shown for the Reynolds number at which the first bubble just formed and

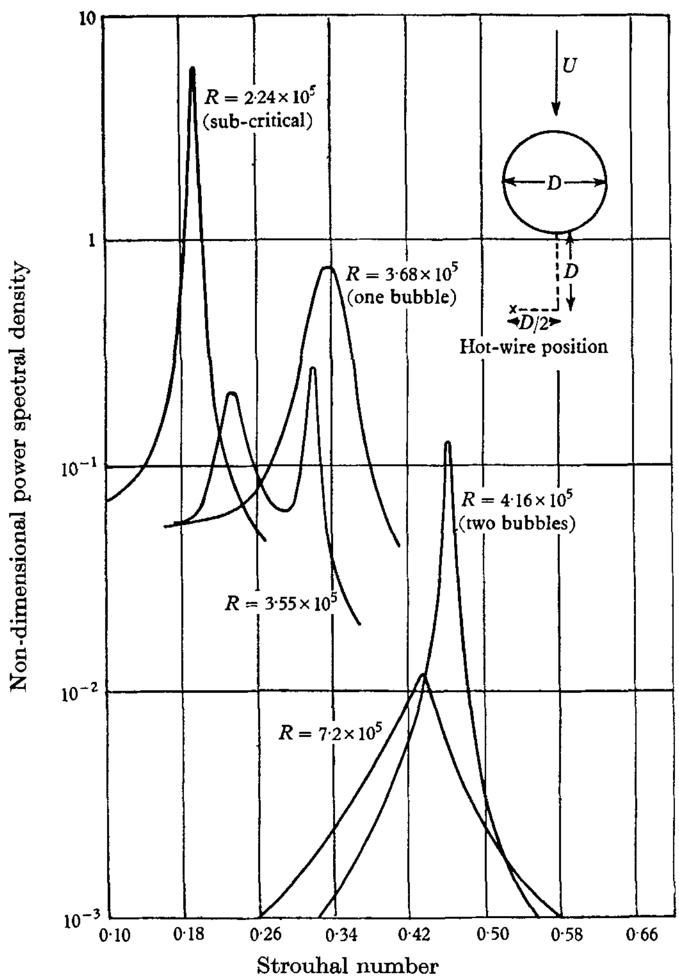

FIGURE 4. Spectra of velocity fluctuations behind a circular cylinder.

exhibits two distinct peaks. The shedding process at this Reynolds number was non-stationary and these peaks did not exist simultaneously. At this particular Reynolds number it was impossible to keep the tunnel speed constant and corrections had to be continually made.

When the first bubble formed, perhaps due to a momentary slight increase in tunnel speed, the drag on the model dropped causing the tunnel speed to increase still further. The reverse procedure occurred when the bubble burst. Thus the two peaks on the spectrum indicate the Strouhal number just before and just after the bubble formed. At Reynolds numbers greater than about

$5{\cdot}5 \times 10^5$ vortex shedding became considerably broader band. The spectrum for $R = 7{\cdot}2 \times 10^5$ is plotted in figure 4. Signals recorded from hot wires positioned either side of the wake at $R = 6 \times 10^5$ are shown in figure 5 (*c*) and clearly illustrate the more random nature of the shedding.

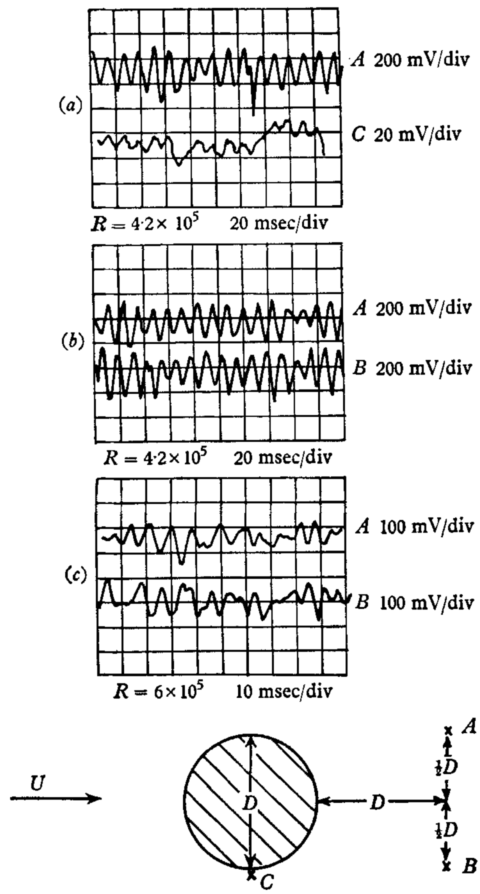

FIGURE 5. Hot-wire signals behind a circular cylinder in the critical Reynolds régime.

4. Discussion of results

As reported in the previous section the vortex shedding frequency in the critical Reynolds number régime was sharply defined. Vortex shedding could be spread across a much wider frequency band, however, by placing any small protuberance on the surface, say a dust particle, or by opening a pressure hole on the cylinder between $\theta \approx 40°$ and $100°$ to atmosphere. Oil flow patterns showed that these disturbances tripped the boundary layer ahead of the laminar separation bubble and locally disrupted it. When this happened the flow was affected over a considerable length of the cylinder and the base pressure along the span was no longer uniform. This was somewhat similar to the behaviour noted by Humphreys (1960). As soon as the base pressure along the span showed any appreciable non-uniformity, sharply defined vortex shedding ceased and the hot-wire signals exhibited a broader-band type of behaviour. Therefore the reason for regular shedding ceasing is due to gross three-dimensionality of the flow caused by turbulent wedges from the front of the cylinder disrupting the separation bubbles. With care, however, regular shedding can continue up to a Reynolds number of at least $5{\cdot}5 \times 10^5$. It is suggested that the disappearance of narrow-band vortex shedding could be taken as a criterion for determining the end of the critical régime.

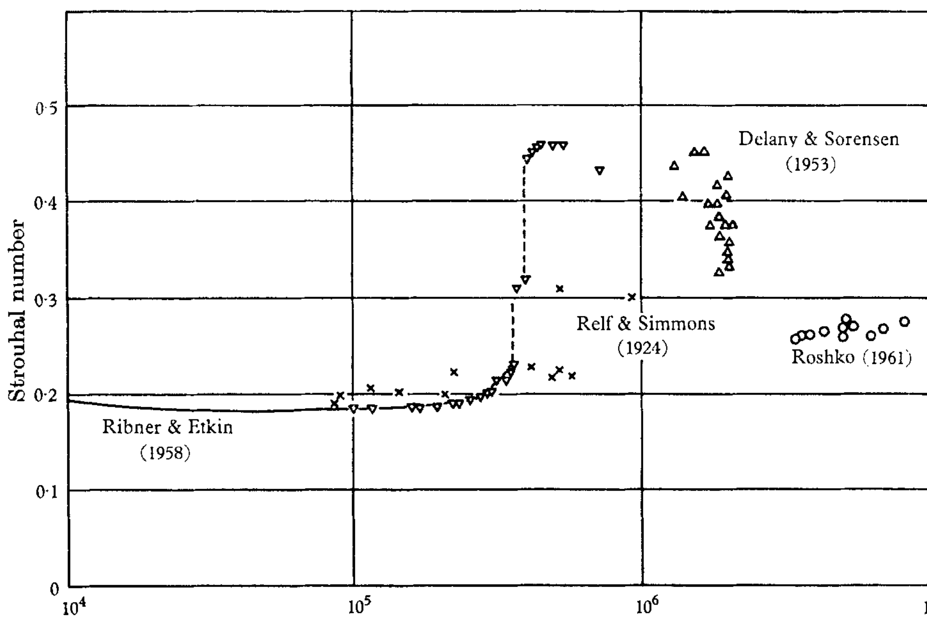

FIGURE 6. Circular cylinders, Strouhal number *versus* Reynolds number. ∇, present results.

The Strouhal numbers measured in this investigation are shown in figure 6, together with other published data (taken from Roshko 1961). The results of Relf & Simmons (1924) are considerably scattered and some of this may be attributable to the quality of flow in their tunnel. It is believed that the accuracy of the present results was considerably higher than that of Relf & Simmons. Of more interest is the agreement between the present results and those of Delany & Sorensen (1953) in the supercritical régime. The Strouhal numbers presented by Delany & Sorenson are based on the 'predominant frequencies encountered'. It is suggested that, in the broad-band shedding régime, the spectral peak will be centred on a Strouhal number between 0·4 and 0·45 over the Reynolds number range 6×10^5 to $1{\cdot}6 \times 10^6$.

5. Conclusions

Regular vortex shedding behind a two-dimensional circular cylinder has been found up to a Reynolds number of $5 \cdot 5 \times 10^5$. When the time-mean drag coefficient is a minimum the Strouhal number has a maximum value of 0·46. Vortex shedding became spread across a wider band of frequencies when turbulent wedges from the front of the cylinder disrupted the laminar separation bubbles.

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